The linear momentum of the cyclic motion is also found to be

$$\pi c^2 \kappa \rho \left\{ 1 - 4(\lambda_0 + 1)\, \delta_0^2 - \frac{4\lambda_0^2 + 5\lambda_0 - 2}{2}\, \delta_0^4, \text{ \&c.} \right\}.$$

A few cases of motion of the ring are then discussed.

6. The annular form of fluid rotating in relative equilibrium is next considered, when the radius of the mean circle of the ring is at least twice as great as the mean radius of the cross-section.

The equation of the cross-section is assumed to be

$$\rho = a\,(1 + \beta_1 \cos\chi + \beta_2 \cos 2\chi + \ldots\ldots),$$

where β_1, β_2, &c., are small quantities.

Taking the centre of gravity of the cross-section as origin, β_1 is seen to be of the 5th order, and it is shown that, as far as the 4th order,

$$\frac{\omega^2}{\pi} = \left(\lambda_0 + \frac{3}{4}\right)\sigma^2 - \frac{1}{8}\left(\lambda_0 + \frac{19}{12}\right)\sigma^4,$$

$$\beta_2 = \frac{\frac{5}{8}\sigma^2 \left\{ \left(\lambda_0 + \frac{7}{12}\right) + \frac{5\sigma^2}{48}\left(\lambda_0 - \frac{107}{120}\right) \right\}}{1 - \left(\lambda_0 + \frac{1}{2}\right)\sigma^2},$$

$$\beta_3 = \frac{5}{128}\left(\lambda_0 - \frac{7}{24}\right)\sigma^3,$$

$$\beta_4 = \frac{75\lambda_0^2 + 90\lambda_0 + 23}{256}\,\sigma^4.$$

The shape of the cross-section is roughly elliptical, the major axis of the ellipse being perpendicular to the axis of revolution.

V. "On the Residues of Powers of Numbers for any Composite Modulus, Real or Complex." By Geoffrey T. Bennett, B.A. Communicated by Professor Cayley, F.R.S. Received April 8, 1892.

(Abstract.)

The present work consists of two parts, with an Appendix to the second. Part I deals with real numbers, Part II with complex.

In the simple cases, when the modulus is a real number which is an odd prime, a power of an odd prime, or double the power of an odd prime, we know that there exist primitive roots of the

modulus: that is, that there are numbers whose successive powers have for their rests the complete set of numbers less than, and prime to, the modulus. A primitive root may be said to generate by its successive powers the complete set of rests. It is also known that in general, when the modulus is any composite number, though primitive roots do not exist, there may be laid down a set of numbers, which will here be called *generators*, the products of powers of which give the complete set of rests prime to the modulus.

The principal object of Part I is to investigate the relations which must subsist among any such set of generators; to determine the most general form that they can take; to show how to form any such set of generators, and, conversely, to furnish tests for the efficiency, as generators, of any given set of numbers. Other results which are obtained as instrumental in effecting these objects, such as the determination of the number of numbers that belong to any exponent, may also possess independent interest.

The object of Part II is to make, for complex numbers, an investigation which shall be as nearly as possible parallel to that of Part I for real numbers. Much of the work of Part I may be applied immediately to complex numbers; of the rest some will need slight modification, and some will need replacing by propositions leading to corresponding results. Of those cases which thus call for independent treatment, the most noticeable is that of the modulus $(1+i)^\lambda$, which is the complex analogue of the real modulus 2^λ.

The work is put in the form of a series of propositions, and is started almost from first principles. The early part is consequently elementary, but the advantages of completeness and ease of reference may be more than sufficient to compensate for this. A large number of illustrative examples are given. These will sometimes, perhaps, assist in elucidating the symbolical proofs which they follow: in all cases they will help to maintain clearly the actual arithmetical meaning of the results arrived at, a meaning which may easily seem obscure if it be noticed only in its symbolical and generalised form.

The Appendix contains tables of indices for complex numbers for all moduli whose norms do not exceed 100.

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